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## Cosmological Redshift-Space Distortion II : distance in an inhomogeneous universe and evolution of bias

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### Abstract

We discuss the two potentially important effects which should be taken into account in the analysis of the cosmological redshift-space distortion especially at high redshifts; the effect of inhomogeneities in the light propagation and the evolution of bias. Although such inhomogeneities affect significantly the degree of the cosmological redshift distortion, one can determine the density parameter  $\Omega_0$  from the distortion pattern at  $z = 0$ , and the cosmological constant  $\lambda_0$  from that at  $z = (1 \sim 2)$ . In particular we show that in low-density ( $\Omega_0 \ll 1$ ) universes one can determine the value of  $\lambda_0$  and even the evolution of the bias parameter almost insensitively to the degree of the clumpiness of the matter distribution in the universe.

*Subject headings:* cosmology: theory — large-scale structure of the universe — methods: statistical

# 1 Introduction

Alcock & Paczyński (1979) proposed a geometrical test for the cosmological constant  $\lambda_0$ . Only very recently this idea attracted much attention, and several authors discussed more specific and realistic methods applicable for samples of galaxies and quasars at high redshifts  $z$  (Phillipps 1994; Ryden 1995; Ballinger, Peacock, & Heavens 1996; Matsubara & Suto 1996, hereafter Paper I).

The angular diameter distance  $D_A$ , which plays a key role in the geometrical test at high  $z$ , depends sensitively on the inhomogeneous matter distribution as well as  $\lambda_0$  and the density parameter  $\Omega_0$ . A reasonably realistic approximation to the light propagation in an inhomogeneous universe is given by Dyer & Roeder (1973). They assume that the fraction  $\alpha$  of the total matter in the universe is distributed smoothly and the rest is in the clumps. If the observed beam of light propagates far from any clump, then the angular diameter distance  $D_A(z; \alpha, \Omega_0, \lambda_0)$  satisfies

$$\frac{d^2 D_A}{dz^2} + \left[ \frac{2}{1+z} + \frac{1}{H(z)} \frac{dH(z)}{dz} \right] \frac{dD_A}{dz} + \frac{3}{2} \frac{\alpha H_0^2 \Omega_0 (1+z)}{H(z)^2} D_A = 0, \quad (1)$$

with  $D_A(z=0) = 0$  and  $dD_A/dz(z=0) = 1/H_0$ , where  $H_0$  is the present Hubble constant ( $\equiv 100 h \text{ km/sec/Mpc}$ ) and  $H(z) \equiv H_0 \sqrt{\Omega_0(1+z)^3 + (1 - \Omega_0 - \lambda_0)(1+z)^2 + \lambda_0}$ . Alcock & Paczyński (1979) and all the above papers in the cosmological redshift-space distortion adopted a *standard* distance which corresponds to an extreme case of  $\alpha = 1$ . The effect of inhomogeneity represented by the parameter  $\alpha$  in the above approximation, however, is quite large for high  $z$  (see Figure 1 below).

In addition, the possible time-dependence of the bias parameter  $b(z)$ , the ratio of the fluctuations of structures relative to those of mass, hampers the observational estimate of  $\lambda_0$  and  $\Omega_0$  through the anisotropies of the power-spectrum (Ballinger, Peacock, & Heavens 1996) and of that of two-point correlation functions (Paper I). In what follows we adopt the linear bias model by Fry (1996):

$$b(z) = 1 + \frac{D(0)}{D(z)}(b_0 - 1), \quad (2)$$

where  $b_0$  is the present value of the bias parameter, and  $D(z)$  is the linear growth rate. In his model, objects are formed at a fixed time by a process of biasing and the subsequent motions are determined by the gravitational potential. The result (2) is a consequence of linear perturbation theory of his model for the universe with arbitrary  $\Omega$  and  $\Lambda$ . Apparently this is one of the simplest possibilities from perturbation consideration, and more realistically  $b(z)$  would be determined by very complicated astrophysical processes. Nevertheless this model serves to highlight the effect of the evolution of bias specifically in the present context.

In this *Letter*, we extend the analysis of Paper I by taking account of the above two effects, and compute the anisotropy of the two-point correlation function in cold dark matter (CDM) models using linear theory.

## 2 Cosmological redshift distortion in an inhomogeneous universe

Let us consider a pair of objects located at redshifts  $z_1$  and  $z_2$  whose redshift difference  $\delta z \equiv z_1 - z_2$  is much less than their mean redshift  $z \equiv (z_1 + z_2)/2$ . Then the observable separations of the pair parallel and perpendicular to the line-of-sight direction are given as  $s_{\parallel} = \delta z/H_0$  and  $s_{\perp} = z\delta\theta/H_0$ , respectively, and  $\delta\theta$  denotes the angular separation of the pair on the sky. The cosmological redshift-space distortion originates from the anisotropic mapping between the redshift-space coordinates,  $(s_{\parallel}, s_{\perp})$ , and the real comoving ones,  $(x_{\parallel}, x_{\perp}) \equiv (c_{\parallel}s_{\parallel}, c_{\perp}s_{\perp})$ ;  $c_{\parallel}(z) = H_0/H(z)$  and  $c_{\perp}(z) = H_0(1+z)D_A/z$ . The cosmological redshift distortion is based on the fact that  $c_{\perp}(z)$  and  $c_{\parallel}(z)$  depend on  $\Omega_0$  and  $\lambda_0$  in a different manner and is characterized by their ratio  $\eta(z) \equiv c_{\parallel}(z)/c_{\perp}(z)$ .

As shown in Figure 1, the effect of inhomogeneity makes larger difference than that of  $\lambda_0$  especially for  $z \gg 1$  (throughout the present paper we compute  $D_A$  using the ANGSI routine provided by Kayser, Helbig, & Schramm 1996). Moreover  $\alpha$  is a stochastic variable in principle, and may be different depending on the line-of-sight direction. Thus  $\alpha$  cannot be predicted theoretically unless one adopts some ad-hoc assumption on the distribution of dark matter. In reality, however, the situation is not so bad. Since the expectation value of  $\alpha$  is determined by the effective volume of the beam of the light bundles, it depends on the depth  $z$  and the angular separation  $\delta\theta$  (of the quasar pair in the present example). For larger  $z$  and larger  $\delta\theta$ ,  $\alpha(z, \delta\theta)$  should approach unity in any case, and the result based on the standard distance as in Paper I would be basically correct. Since we do not have any justifiable model for  $\alpha(z, \delta\theta)$ , we will consider two extreme cases  $\alpha(z, \delta\theta) = 1$  (filled beam) and 0 (empty beam). Our main purpose here is to highlight the importance of the effect even though more realistically  $\alpha(z, \delta\theta)$  is somewhere in between the two extreme cases and  $\alpha(z, \delta\theta) = 1$  is a qualitatively reasonable guess especially for  $z \gg 1$  and  $\delta\theta \gg 1$ .

It is quite reassuring that even in these extreme cases the inhomogeneity effect is much weaker than that of  $\lambda_0$  at  $z \lesssim 2$  in *low density* universes as the right panels in Figure 1 illustrate clearly. Since most observational evidence points to a relatively low value of  $\Omega_0$  around  $(0.1 \sim 0.3)$  (e.g., Peebles 1993; Suto 1993), this suggests that the optimal redshift to determine  $\lambda_0$  in low  $\Omega_0$  universes is  $z = (1 \sim 2)$ ; the redshift distortion of galaxy correlation functions at  $z \ll 1$  (Kaiser 1987; Hamilton 1992) is a good probe of  $\Omega_0$  insensitive to  $\lambda_0$ .

## 3 Anisotropies in two-point correlation functions

The relation between the two-point correlation functions of quasars in redshift space,  $\xi^{(s)}(s_{\perp}, s_{\parallel})$ , and that of *mass* in real space  $\xi^{(r)}(x)$  can be derived in linear theory (Paper

I; Hamilton 1992):

$$\xi^{(s)}(s_{\perp}, s_{\parallel}) = \left(1 + \frac{2}{3}\beta(z) + \frac{1}{5}[\beta(z)]^2\right) \xi_0(x)P_0(\mu) - \left(\frac{4}{3}\beta(z) + \frac{4}{7}[\beta(z)]^2\right) \xi_2(x)P_2(\mu) + \frac{8}{35}[\beta(z)]^2 \xi_4(x)P_4(\mu), \quad (3)$$

where

$$\beta(z) \equiv -\frac{1}{b(z)} \frac{d \ln D(z)}{d \ln(1+z)}, \quad \xi_{2l}(x) = \frac{b^2(z)}{2\pi^2} \int_0^\infty dk k^2 j_{2l}(kx) P(k; z), \quad (4)$$

$j_n$ 's are the spherical Bessel functions,  $x \equiv \sqrt{c_{\parallel}^2 s_{\parallel}^2 + c_{\perp}^2 s_{\perp}^2}$ ,  $\mu \equiv c_{\parallel} s_{\parallel} / x$ ,  $P_n$ 's are the Legendre polynomials.

Thus  $\xi^{(s)}(s_{\perp}, s_{\parallel})$  in linear theory crucially depends on the power spectrum of the mass fluctuations  $P(k; z)$ , the bias parameter for quasars  $b(z)$ , and  $\alpha$  as well as  $\Omega_0$  and  $\lambda_0$ . Clearly it seems very difficult to break the degeneracy and reliably determine  $\lambda_0$ , for instance, without further knowledges or assumptions of the other parameters. In principle, one may resort to the extensive curve fits to the observed correlation functions at different scales  $s$  and redshifts  $z$  to estimate the set of parameters. However there is an optimal range of different  $z$  to determine each parameter.

Figure 2 shows the evolution of bias (eq.[2]; upper panels) and of the resulting  $\beta(z)$  parameter (lower panels). This implies that as long as Fry's model of  $b(z)$  is adopted, one can distinguish the value of  $\lambda_0$  independently of the evolution of bias *only in low density* ( $\Omega_0 \ll 1$ ) *models and at intermediate redshifts* ( $z \lesssim 2$ ). Together with the indication from Figure 1 (§2),  $z = (1 \sim 2)$  would be an optimal regime to probe  $\lambda_0$  at least in low-density universes. Figure 3 illustrates the extent to which this is feasible simply on the basis of the anisotropy parameter  $\xi_{\parallel}^{(s)}(s)/\xi_{\perp}^{(s)}(s)$  as a function of  $z$ , adopting the power spectrum of the CDM models; in  $\Omega_0 = 1$  models the value of  $\alpha$  completely changes the  $z$ -dependence of the anisotropy parameter while  $\Omega_0 = 0.1$  models are fairly insensitive to it. In addition,  $\xi_{\parallel}^{(s)}(s)/\xi_{\perp}^{(s)}(s)$  for  $z \lesssim 2$  in  $\Omega_0 = 0.1$  models is basically determined by the biasing parameter at  $z = 0$  and less affected by the evolution of  $b(z)$ . Figure 4 shows the scale-dependence of the anisotropy parameter in  $\Omega_0 = 0.1$  and  $h = 0.7$  CDM models. This clearly indicates that one can distinguish the different  $\lambda_0$  and bias models by analysing the anisotropy of the correlation function at  $z = 1$  almost independently of  $\alpha$ .

## 4 Conclusions

While the importance of the effect of inhomogeneities in the light propagation (e.g., Dyer & Roeder 1973; Kayser, Helbig, & Schramm 1996) is quite well recognized in the study of the gravitational lensing, for instance, it has not attracted particular attention in other fields in observational cosmology until very recently. This would be primarily because this effect

becomes important only at  $z \gtrsim 1$  where previous redshift surveys do not yet provide good statistical samples for the accurate determination of the cosmological parameters. This effect is important also in estimating  $H_0$  and  $q_0$ , using the SN Ia at  $z \gtrsim 0.5$  (Kantowski, Vaughan & Branch 1995; Goobar & Perlmutter 1995) and the Sunyaev - Zel'dovich effect (Kobayashi, Sasaki & Suto 1996).

We have shown that such inhomogeneities, which were neglected in the original proposal by Alcock & Paczyński (1979), also affect significantly the degree of the cosmological redshift distortion, and in fact hampers the accurate estimate of the cosmological constant especially in  $\Omega_0 = 1$  universes. In low-density universes, however, one can determine  $\lambda_0$  in principle by optimizing the range of  $z$  (see Fig.1);  $\Omega_0$  (or  $\beta$  parameter) is best determined from the redshift-space clustering at  $z = 0$  (Kaiser 1987; Hamilton 1992). Once  $\Omega_0$  is fixed, the cosmological redshift distortion around  $z \sim 1$  is most sensitive to the value of  $\lambda_0$ . Then the distortion at the higher redshift provides clue to the degree of the clumpiness or  $\alpha$  which in fact would be also a function of  $z$ . Since either  $\alpha = 0$  or  $\alpha = 1$  is an extreme case, the realistic situation would be more favorable to the determination of  $\Omega_0$  and  $\lambda_0$ . Nevertheless the degree of the clumpiness is another important factor which should be always kept in mind in the analysis of the clustering of structures at high redshifts.

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## REFERENCES

- Alcock, C. & Paczyński, B. 1979, *Nature*, 281, 358
- Ballinger, W.E., Peacock, J.A. & Heavens, A.F. 1996, *MNRAS*, in press.
- Dyer, C.C. & Roeder, R.C. 1973, *ApJL*, 180, L31
- Fry, J. 1996, *ApJ*, 461, L65
- Goobar, A. & Perlmutter, S. 1995, *ApJ*, 450, 14
- Hamilton, A.J.S. 1992, *ApJL*, 385, L5
- Kaiser, N. 1987, *MNRAS*, 227, 1
- Kantowski, R., Vaughan, T., & Branch, D. 1995, *ApJ*, 447, 35
- Kayser, R., Helbig, P., & Schramm, T. 1996, *A&A*, in press.
- Kobayashi, S., Sasaki, S., & Suto, Y. 1996, *ApJ*, submitted.
- Matsubara, T. & Suto, Y. 1996, *ApJL*, in press (Paper I).
- Peebles, P.J.E. 1993, *Principles of Physical Cosmology* ( Princeton University Press: Princeton)
- Phillipps, S. 1994, *MNRAS*, 269, 1077
- Ryden, B. 1995, *ApJ*, 452, 25
- Suto, Y. 1993, *Prog.Theor.Phys.*, 90, 1173

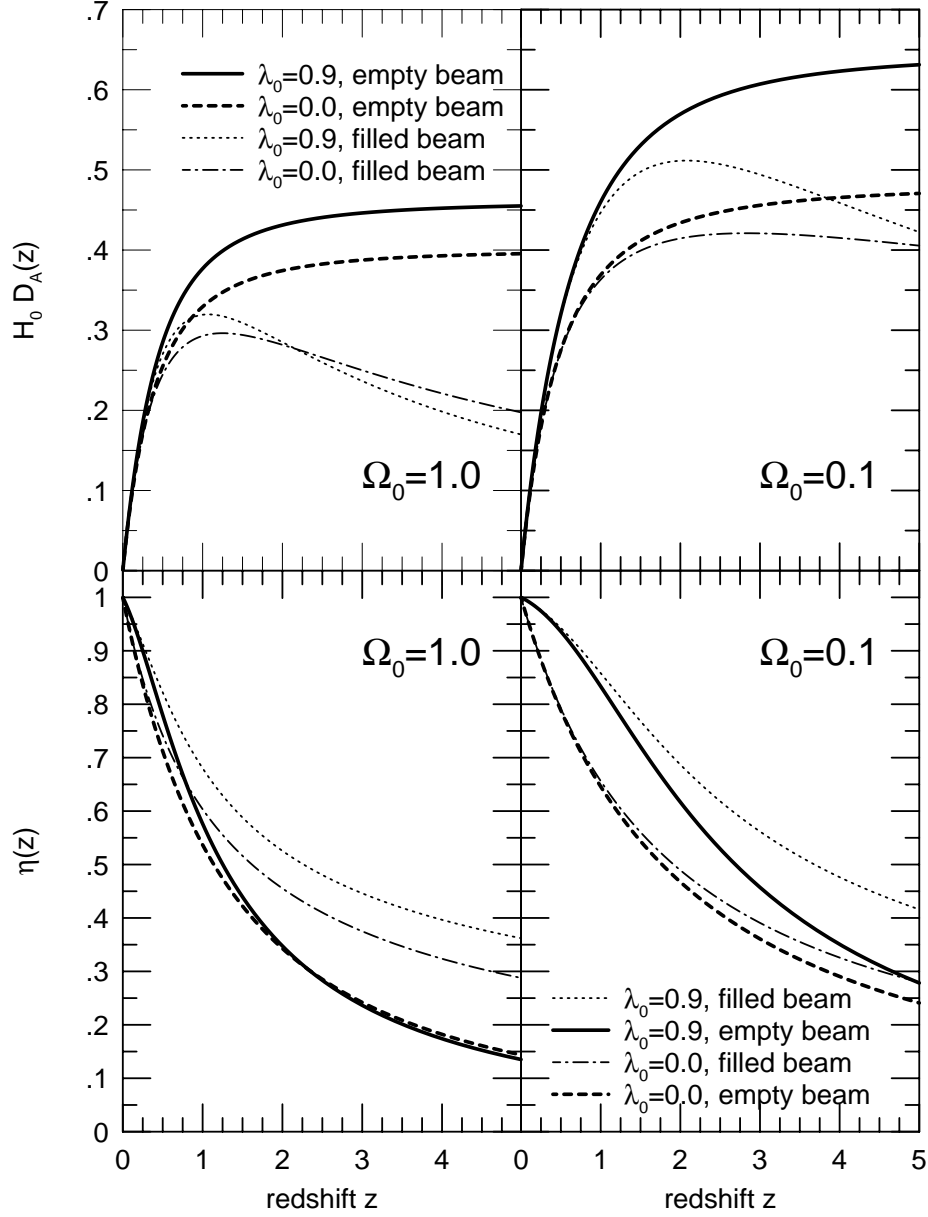


Figure 1: Effect of inhomogeneity on the angular diameter distance  $D_A(z)$  (upper panels) and the correction factor  $\eta(z)$  (lower-panels) for  $\lambda_0 = 0$  and  $0.9$  models in  $\Omega_0 = 1$  (left panels) and  $\Omega_0 = 0.1$  (right panels) universes. Thick lines indicate the results for the empty beam ( $\alpha = 0$ ), while thin lines for the filled beam ( $\alpha = 1$ ).

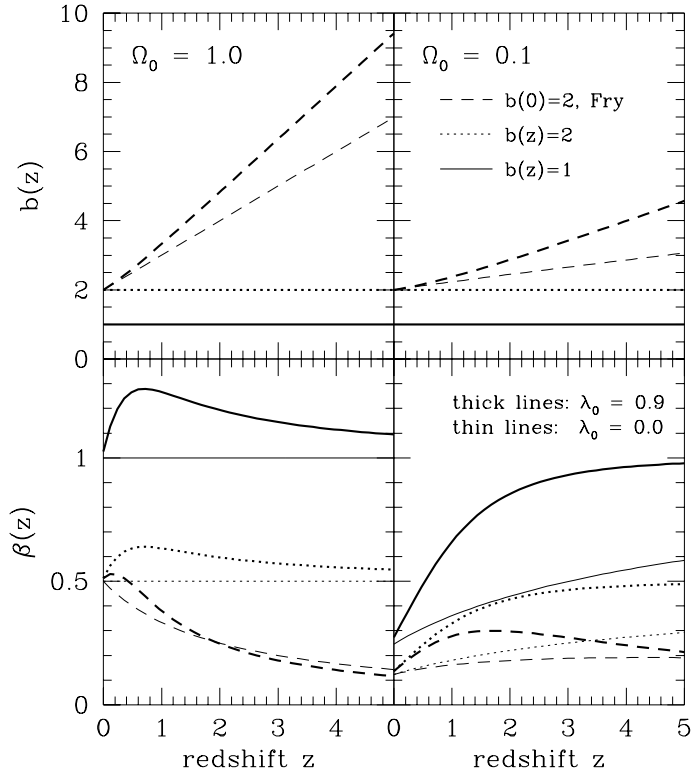


Figure 2: Evolution of bias and  $\beta(z)$ . Dashed and solid lines assume constant biasing parameter  $b(z) = 2$  and 1, respectively, while dotted lines adopt the evolution model of Fry (1996) with  $b = 2$  at  $z = 0$ . Thick and thin lines correspond to  $\lambda_0 = 0.9$  and 0, respectively.



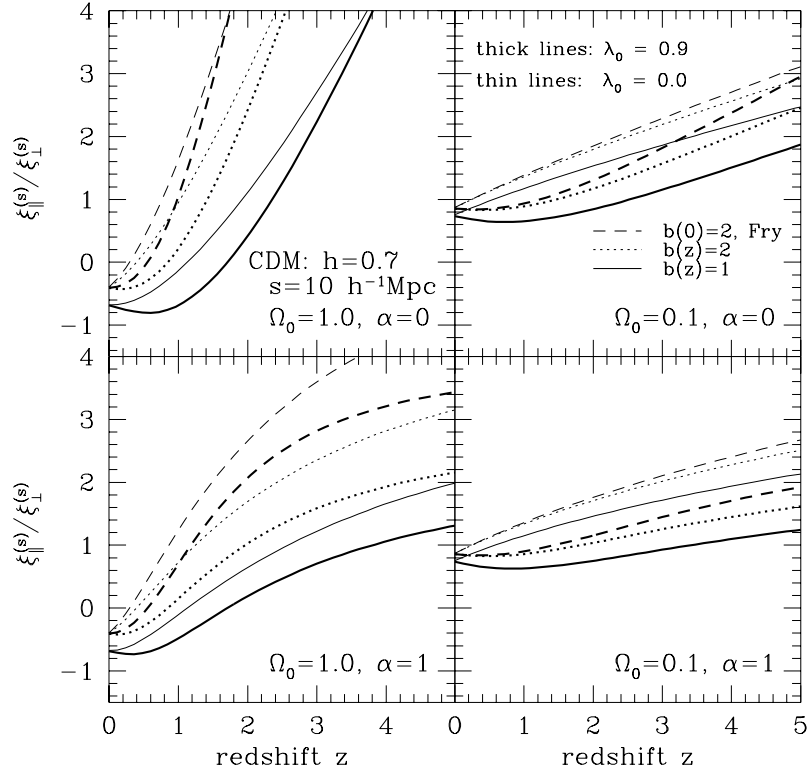


Figure 3: The anisotropy parameter  $\xi_{\parallel}^{(s)}(s)/\xi_{\perp}^{(s)}(s)$  as a function of  $z$  at  $s = 10h^{-1}\text{Mpc}$  in cold dark matter universes with  $H_0 = 70\text{km/sec/Mpc}$ . Upper-panels assume  $\alpha = 0$  (empty beams), while lower-panels  $\alpha = 1$  (filled beams).

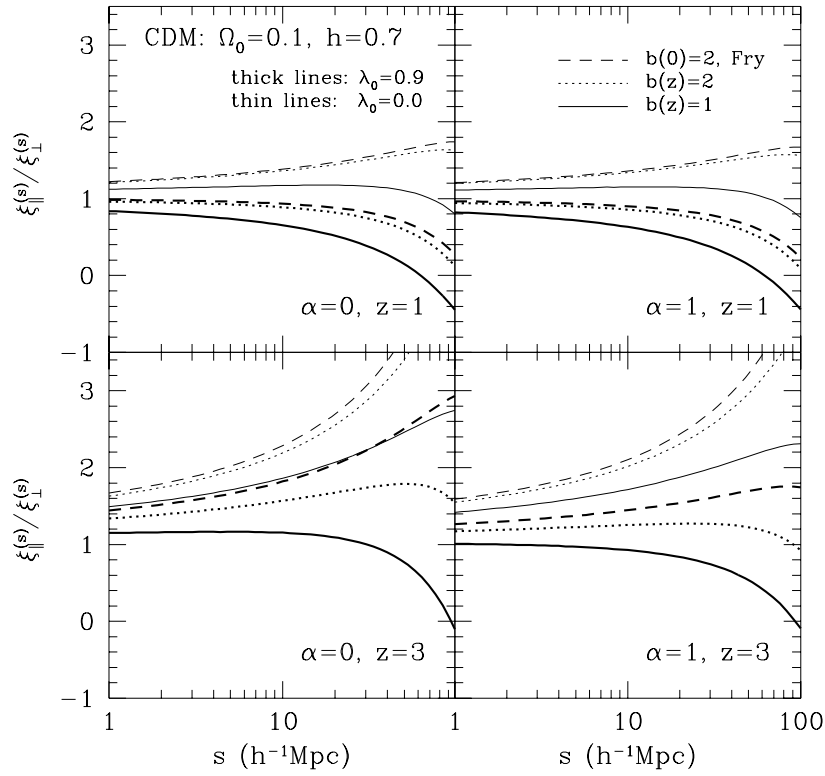


Figure 4: The anisotropy parameter  $\xi_{||}^{(s)}/\xi_{\perp}^{(s)}$  as a function of  $s$  at  $z = 1$  (upper-panels) and at  $z = 3$  (lower-panels) in cold dark matter universes with  $H_0 = 70\text{km/sec/Mpc}$ . Left-panels assume  $\alpha = 0$  (empty beams), while right-panels  $\alpha = 1$  (filled beams).